

## Core Functions and Operations

### Polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$n =$  non-negative whole number  $(0, 1, 2, \dots)$

$a_0, a_1, \dots, a_n =$  constant real numbers (coefficients)

$a_n \neq 0 \Rightarrow \deg(f) = n$  (degree of  $f$ )

degree 0 = constant functions  $f(x) = a_0$

degree 1 = linear (non-constant) functions

$$f(x) = a_1 x + a_0$$

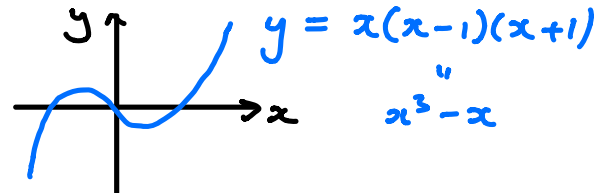
degree 2 = Quadratic functions

$$f(x) = a_2 x^2 + a_1 x + a_0$$

Graph is parabola.

degree 3 = Cubic functions

E.g.



$f(x)$  polynomial  $\Rightarrow$  domain is all real numbers

Odd degree  $\Rightarrow$  Range is all real numbers

Even degree  $\Rightarrow$  No simple answer. Depends

on polynomial. E.g.

Range of  $x^2 + 1 = [1, \infty)$

Range of  $1 - x^4 = (-\infty, 1]$

### Power Functions

$$f(x) = x^a \quad (a \text{ is any real number})$$

$n =$  non-negative whole number  $(0, 1, 2, \dots)$

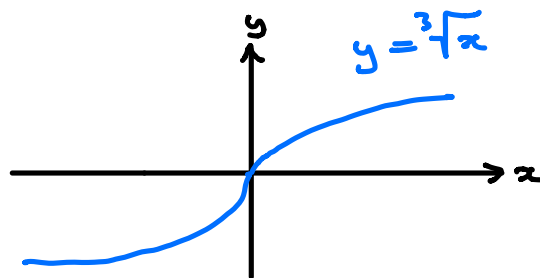
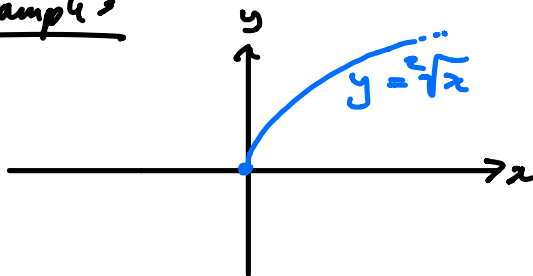
$a = n \Rightarrow f(x)$  polynomial

$a = \frac{1}{n} \Rightarrow f(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$  (Notation  $\sqrt{\quad} = \sqrt{\quad}$ )

$n$  odd  $\Rightarrow$  domain / range all real numbers

$n$  even  $\Rightarrow$  domain / range is all non-negative real numbers.

Example



## Operations on Functions

We can  $+$ ,  $-$ ,  $\times$  and  $\div$  functions

Examples

1.  $f(x) = x^2 + x - \sqrt{x}$

2.  $f(x) = \frac{2x + 7x^5}{x^5}$

Rational Function = Ratio of two polynomials  
$$\frac{p(x)}{q(x)}$$

Domain of  $\frac{p(x)}{q(x)} =$  All  $x$  such that  $q(x) \neq 0$ .

Example  $f(x) = \frac{x^2 + x + 1}{x^2 + x - 2}$  has domain  $x \neq 1, -2$

We can also compose functions (ie do one then the other)

$f, g$  functions.

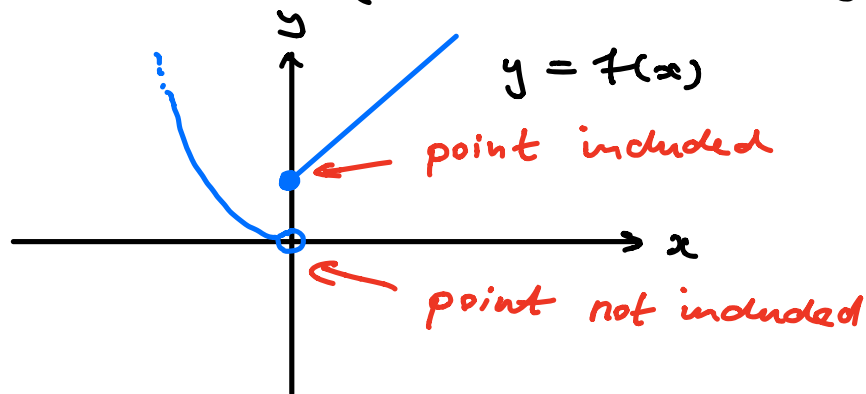
$$f \circ g(x) = f(g(x)) \quad \leftarrow \text{do } g \text{ first then } f.$$

↑  
Notation for composition of  $g$  with  $f$ .

Example  $\sqrt[5]{x^2+x+1} = f(g(x))$  where  
 $g(x) = x^2+x+1$ ,  $f(x) = \sqrt[5]{x}$ .

Piecewise Function = Function given by different formulae on different intervals.

Example  $f(x) = \begin{cases} x+1 & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$



# Translations and Reflections :

$c \geq 0$  ,  $f(x)$  function

